Study upon the Unsymmetrical Condition of the Induction Machine by Using *Representative Rotational Phase Vectors*

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***Abstract*— On the basis of the mathematical model, called in a previous paper *in total fluxes*, and which is proper for the analysis of transient operation of the two-phase induction machine, one obtains the symmetrical steady-state equations, which are valid for three-phase machines, as well. The obtained mathematical expressions are much more simple and easier to be used that the consecrated ones which are generally applied in scientific literature. Moreover, considerations are to be made upon the space-time rotational vectors, emphasizing their importance in understanding the physical phenomena that characterize induction machines. The use of these space vectors is further tested out for the study of unsymmetrical supply which gives a much faster method in obtaining the electromagnetic torque expression. Finally, the results are compared with the ones that come out from the traditional methods.**

***Index Terms*—induction machine, unsymmetrical condition, representative rotational phase vector**

I. INTRODUCTION

In support of our approach, defining of the space-time rotational vectors, which were presented under similar formulations in scientific literature [1-8], is firstly necessary. The superposing effect as regards the quantities of the electric field determined by the two-phase supply system and the corresponding magnetic field is also considered as a prior probability. The supply voltages are applied along the turns placed in the slots (collinear to *Oz* axis). If the winding is placed in the slots according to a sinusoidal law, *one decide that the applied voltage phase vector, u* to be represented as a segment in the *xOy* space, orientated towards positive axis of the winding. The length of this segment is maximum when the applied voltage is maximum, too. Ideally, in default of magnetic leakages and drop voltages corresponding to winding resistance, the magnetic fields (more precisely, *the total fluxes* ψ which are preponderantly effective for the real cylindrical machines) close in a radial pattern inside the *xOy* space. It has a harmonic distribution on the periphery (on the circle that matches the middle of the air-gap) which means that the *flux density* has maximum values in the centrum of the supplied winding only when *the applied voltage reaches the 0 value* (according to induced voltage law: $u=d\psi/dt$). For our demonstration, the stator phase vectors of the voltages and total fluxes will be represented in the *xOy* space for different but consecutive moments according to Leblanc theorem (any magnetic flux of ψ_m amplitude, which is created by a winding with cosine distribution and single-phase feeding, is equivalent with two rotating and equal fluxes with the amplitude of $(1/2)\psi_m$ but which rotates in opposite directions with coequal speed – *forward* and *backward* traveling waves respectively), [9-14].

II. THE REPRESENTATIVE PHASE VECTORS OF THE IDEAL INDUCTION MACHINE

The case of *ideal machine with no leakage fluxes zero value winding resistances* is taken into discussion. The following symbolic notations are used:

$$\begin{aligned} &u_{as} = U_{as}\sqrt{2}\cos\omega_s t \leftrightarrow \underline{U}_{as} = U_{as}\sqrt{2}(e^{j\omega_s t} + e^{-j\omega_s t})/2 = \underline{U}_{asd} + \underline{U}_{asi};\\ &\underline{U}_{asd} = \sqrt{2}\frac{U_{as}}{2}e^{j\omega_s t};\quad \underline{U}_{asi} = \sqrt{2}\frac{U_{as}}{2}e^{-j\omega_s t}; \end{aligned} \tag{1}$$

$$\begin{aligned} &\psi_{as} = \frac{U_{as}}{\omega_s}\sqrt{2}\cos\left(\omega_s t - \frac{\pi}{2}\right) \leftrightarrow \underline{\Psi}_{as} = \frac{\Psi_{as}}{2}\sqrt{2}\left[e^{j(\omega_s t-\pi/2)} + e^{-j(\omega_s t-\pi/2)}\right] = \underline{\Psi}_{asd} + \underline{\Psi}_{asi}\\ &\underline{\Psi}_{asd} = \sqrt{2}\frac{\Psi_{as}}{2}e^{j(\omega_s t-\pi/2)} = -\frac{j}{\omega_s}\underline{U}_{asd};\underline{\Psi}_{asi} = \sqrt{2}\frac{\Psi_{as}}{2}e^{-j(\omega_s t-\pi/2)} = \frac{j}{\omega_s}\underline{U}_{asi};\Psi_{as} = \frac{U_{as}}{\omega_s} \end{aligned} \tag{2}$$

$$\begin{aligned} &u_{bs} = U_{bs}\sqrt{2}\cos(\omega_s t - \pi/2) \leftrightarrow \underline{U}_{bs} = \frac{U_{bs}}{2}\sqrt{2}\left[e^{j(\omega_s t-\pi/2)} + e^{-j(\omega_s t-\pi/2)}\right] = \underline{U}_{bsd} + \underline{U}_{bsi};\\ &\underline{U}_{bsd} = -j\frac{U_{bs}}{U_{as}}\underline{U}_{asd};\underline{U}_{bsi} = j\frac{U_{bs}}{U_{as}}\underline{U}_{asi}, \end{aligned} \tag{3}$$

$$\psi_{bs}=\frac{U_{bs}}{\omega_s}\sqrt{2}\cos(\omega_s t-\pi)\leftrightarrow \underline{\Psi}_{bs}=\frac{\Psi_{bs}}{2}\sqrt{2}\left[e^{j(\omega_s t-\pi)}+e^{-j(\omega_s t-\pi)}\right]=\underline{\Psi}_{bsd}+\underline{\Psi}_{bsi};$$
$$\underline{\Psi}_{bsd}=-\sqrt{2}\frac{\Psi_{bs}}{2}e^{j\omega_s t};\quad \underline{\Psi}_{bsi}=-\sqrt{2}\frac{\Psi_{bs}}{2}e^{-j\omega_s t};\Psi_{bs}=\frac{U_{bs}}{\omega_s}. \tag{4}$$

If one consider a cross section of a symmetrical two-phase machine (and more precisely a path along mid-stator cylinder) and the winding axes have the orientation *as-Ox* and *bs-Oy* respectively, then the phase vectors of the voltages and total fluxes, for different moments, have the positions indicated in Fig. 1 (where a two-phase symmetrical condition is assumed, $U_{as}=U_{bs}$).

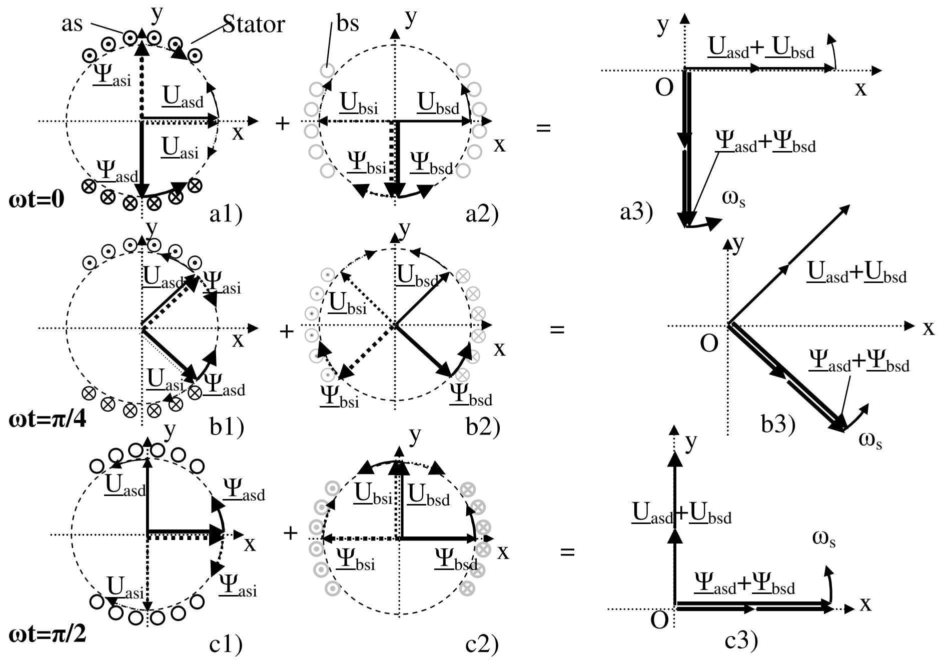

Fig. 1 Representative rotational phase vectors of two-phase induction machine

Fig. 1 presents the voltage and total flux phase vectors corresponding to *forward* and *backward* order for three different moments: $\omega t=0$ - Fig. 1a1)-a3), $\omega t=\pi/4$ - Fig. 1b1)-b3) and $\omega t=\pi/2$ - Fig. 1c1)-c3). In contrast with "classical" representation manner, where the *cross* and *dot* signs placed inside a turn show the orientation of the flowing current, in our case the symbols define the polarity of applied voltage across turns (to avoid any confusion has to be mentioned that the purely inductive circuits have a time alteration of phase of $\pi/2$ between current and applied voltage).

During the considered interval (a quarter period) the applied voltage on *as* phase starts from maximum value and decreases to zero value (see Fig.1-a1,b1,c1) while the voltage corresponding to *bs* phase starts from zero value and increases to maximum value (Fig.1-a2,b2,c2). The resultant values of voltage and total flux phase vectors come from a geometrical summation of the forward components corresponding to the two phases of the machine. These components are coequal and collinear. On the contrary, the backward components are coequal but in opposite directions for every moment and consequently the sum is always zero. For example, the resultant phase vector of total stator flux has a constant absolute value and the rotation angle is of $\pi/2$ rad. (during a quarter period, the apex of the resultant phase vector covers a quarter of the circle inside the plane *xOy*). This is a coincidence that allows a reciprocal conversion of the temporal and spacial angles and the phase vectors (obtained by means of *analytical representation*) from the complex space (+1,+j) can be assimilated to phase vectors in *xOy* space, which will be denoted *representative vectors*.

To express in an analytical manner the *representative vector* of total stator fluxes is necessary a geometrical summation of the components corresponding to *Ox* axis (the $\bar{i}$ versor) and *Oy* axis (the $\bar{j}$ versor) of the two windings, *as* and *bs*. Along *Ox* axis, the total flux is created by *as* winding with a cosinusoidal time variation and an initial phase of $-\pi/2$. The same assumption for *bs* phase but acting on *Oy* axis with an initial phase of $-\pi$.

$$\overline{\Psi}_{sR}=\psi_{asx}\bar{i}+\psi_{bsy}\bar{j}=\Psi_{as}\sqrt{2}\cos\left(\omega_s t-\frac{\pi}{2}\right)\bar{i}+\Psi_{as}\sqrt{2}\cos\left(\omega_s t-\frac{\pi}{2}-\frac{\pi}{2}\right)\bar{j}; \tag{5}$$

This vector can be expressed in a different way (polar coordinates) as $\overline{\Psi}_{sR}=\Psi_{sR}\angle\theta$. The absolute value of this vector is $\Psi_{sR}=\sqrt{\psi_{asx}^2+\psi_{bsy}^2}=\Psi_{as}\sqrt{2}$ -constant, and the argument $\theta=\omega_s t-\pi/2$, which is time dependent. The angular speed comes from phase derivative $\Omega_s=d\theta/dt=\omega_s$ and is equal to applied voltage pulsation. Notable is the fact that *the projections of the representative vector along Ox and Oy axes have coequal length with instantaneous values* of the total fluxes created by the *as* and *bs* windings. This is a strong motive in adopting the concept of *representative space-time vector* of the stator flux. It must be also mentioned that the representative vector of the stator flux shows any moment the radial direction of the cross section plane where the density of stator magnetic flux lines (with radial air-gap flux density) is maximum.

A different way to define the *representative space-time* vector of the stator flux is based on one-to-one correspondence between *xOy* space ($\bar{i}$, $\bar{j}$ versors) and complex space *(+1,+j)*. The following statement is allowed: the $\bar{j}$ versor can be obtained by rotating in the same plane the versor $\bar{i}$ with $\pi/2$ which is equivalent to "multiplication by *j*" in "simplified complex" approach. One can define:

$$\overline{\Psi}_{sR} = \psi_{asx}\bar{i} + \psi_{bsy}\bar{j} \leftrightarrow \underline{\Psi}_{sR} = \underline{\Psi}_{as} + e^{j\delta}\underline{\Psi}_{bs}; \delta = \pi/2 \quad (6)$$

The angle $\delta = \pi/2$ has the signification of a *spacial angle* between the machine windings. Taking into consideration the forward and backward components presented above, one obtain:

$$\begin{aligned}\underline{\Psi}_{sR} &= \underline{\Psi}_{as} + j\underline{\Psi}_{bs} = \underline{\Psi}_{asd} + \underline{\Psi}_{asi} + j\underline{\Psi}_{bsd} + j\underline{\Psi}_{bsi} = \frac{\Psi_{as}}{2}\sqrt{2}\Big[e^{j(\omega_s t-\pi/2)} + e^{-j(\omega_s t-\pi/2)} + \\ &+ je^{j(\omega_s t-\pi/2-\pi/2)} + je^{-j(\omega_s t-\pi)}\Big] = \Psi_{as}\sqrt{2}\, e^{j(\omega_s t-\pi/2)}\end{aligned} \quad (7)$$

which prove in an analytical way that there is a summation of the forward components and a subtraction of backward components which nullify them.

Observation: The direction of rotation of the representative vector can be reversed if the polarity of the supply voltage for one winding is reversed as well. For example, if the reversed phase is *b-y*, then $\underline{\Psi}_{bs}$ is reversed and the representative vector becomes:

$$\begin{aligned}\underline{\Psi}'_{sR} &= \underline{\Psi}_{as} + j(-\underline{\Psi}_{bs}) = \underline{\Psi}_{asd} + \underline{\Psi}_{asi} - j\underline{\Psi}_{bsd} - j\underline{\Psi}_{bsi} = \frac{\Psi_{as}}{\sqrt{2}}\Big[e^{j(\omega_s t-\pi/2)} + e^{-j(\omega_s t-\pi/2)} + \\ &- je^{j(\omega_s t-\pi/2-\pi/2)} - je^{-j(\omega_s t-\pi)}\Big] = \Psi_{as}\sqrt{2}\, e^{-j(\omega_s t-\pi/2)}\end{aligned} \quad (8)$$

In this case, the forward components annihilate each other and the backward components add up.

In conclusion, for the symmetrically fed two-phase induction machine, the representative vector of the resultant stator flux has the length coequal to the amplitude of total flux generated by one phase (*as* or *bs*), $\Psi_{sR} = U_{as}\sqrt{2}/\omega_s$ (its apex covers a circle), the rotation speed is constant and equal to pulsation of applied voltage ω_s, and the direction of rotation is conditioned by the initial phase angle of the two applied voltages. In anticipation, has to be revealed that the representative vector is given by the summation of two opposite rotating vectors, a forward component and a backward one.

III. THE REPRESENTATIVE PHASE VECTORS FOR UNSYMMETRICAL TWO-PHASE CONDITION

The study take into discussion the unsymmetrical supply when $U_{as} \neq U_{bs}$. An analytical approach is possible if the following expressions are used:

$$\begin{aligned}
&u_{as} = U_{as}\sqrt{2}\cos\omega_s t \leftrightarrow \underline{U}_{as} = \frac{1}{2}U_{as}\sqrt{2}(e^{j\omega_s t} + e^{-j\omega_s t}) = \underline{U}_{asd} + \underline{U}_{asi};\\
&\underline{U}_{asd} = \sqrt{2}\frac{U_{as}}{2}e^{j\omega_s t};\quad \underline{U}_{asi} = \sqrt{2}\frac{U_{as}}{2}e^{-j\omega_s t};\\
&\psi_{as} = \frac{U_{as}}{\omega_s}\sqrt{2}\cos\left(\omega_s t - \frac{\pi}{2}\right) \leftrightarrow \underline{\Psi}_{as} = \frac{\Psi_{as}}{2}\sqrt{2}\Big[e^{j(\omega_s t-\pi/2)} + e^{-j(\omega_s t-\pi/2)}\Big] = \underline{\Psi}_{asd} + \underline{\Psi}_{asi}\\
&\underline{\Psi}_{asd} = \sqrt{2}\frac{\Psi_{as}}{2}e^{j(\omega_s t-\pi/2)} = -\frac{j\underline{U}_{asd}}{\omega_s}; \underline{\Psi}_{asi} = \sqrt{2}\frac{\Psi_{as}}{2}e^{-j(\omega_s t-\pi/2)} = \frac{j\underline{U}_{asi}}{\omega_s}; \Psi_{as} = \frac{U_{as}}{\omega_s}\\
&u_{bs} = \lambda U_{as}\sqrt{2}\cos(\omega_s t - \pi/2 + \varepsilon) = U_{b1}\sqrt{2}\cos(\omega_s t - \pi/2) + U_{b2}\sqrt{2}\cos\omega_s t \leftrightarrow\\
&\leftrightarrow \underline{U}_{b1d} + \underline{U}_{b1i} + \underline{U}_{b2d} + \underline{U}_{b2i} = \frac{U_{b1}}{\sqrt{2}}\Big[e^{j(\omega_s t-\pi/2)} + e^{-j(\omega_s t-\pi/2)}\Big] + \frac{U_{b2}}{\sqrt{2}}\Big[e^{j\omega_s t} + e^{-j\omega_s t}\Big],
\end{aligned} \quad (9)$$

where: $U_{b1} = \lambda U_{as}\cos\varepsilon$; $U_{b2} = \lambda U_{as}\sin\varepsilon$. A different form to express the above equtions is

$$\begin{aligned}
u_{bs} &= \lambda U_{as}\sqrt{2}\cos(\omega_s t - \pi/2 + \varepsilon) \leftrightarrow \frac{\lambda U_{as}}{\sqrt{2}}\Big[e^{j(\omega_s t)}e^{j(\varepsilon-\pi/2)} + e^{-j(\omega_s t)}e^{-j(\varepsilon-\pi/2)}\Big] =\\
&= \frac{\lambda U_{as}}{\sqrt{2}}\Big[e^{j\omega_s t}e^{j\varepsilon}(-j) + e^{-j\omega_s t}e^{-j\varepsilon}j\Big] = -j\underline{\lambda}_d\underline{U}_{asd} + j\underline{\lambda}_i\underline{U}_{asi},\quad \underline{\lambda}_d = \lambda e^{j\varepsilon}, \underline{\lambda}_i = \lambda e^{-j\varepsilon}
\end{aligned} \quad (10)$$

$$\begin{aligned}
\psi_{bs} &= \frac{\lambda U_{as}}{\omega_s}\sqrt{2}\cos(\omega_s t - \pi + \varepsilon) \leftrightarrow \underline{\Psi}_{bs} = \frac{\Psi_{b1}}{\sqrt{2}}\Big[e^{j(\omega_s t-\pi)} + e^{-j(\omega_s t-\pi)}\Big] +\\
&+ \frac{\Psi_{b2}}{\sqrt{2}}\Big[e^{j(\omega_s t-\pi/2)} + e^{-j(\omega_s t-\pi/2)}\Big] = \underline{\Psi}_{b1d} + \underline{\Psi}_{b1i} + \underline{\Psi}_{b2d} + \underline{\Psi}_{b2i}; \Psi_{b1} = \frac{U_{b1}}{\omega_s}; \Psi_{b2} = \frac{U_{b2}}{\omega_s}.
\end{aligned} \quad (11)$$

Using the above reasoning, one can define the space-time stator phase vector:

$$\overline{\Psi}_{sR} = \psi_{asx}\bar{i} + \psi_{bsy}\bar{j} \leftrightarrow \underline{\Psi}_{sR} = \underline{\Psi}_{as} + e^{j\delta}\underline{\Psi}_{bs}; \delta = \pi/2 \quad (12)$$

Taking into consideration the forward and backward components above designated, one obtain:

$$\begin{aligned}\underline{\Psi}_{sR} &= \underline{\Psi}_{as} + j\underline{\Psi}_{bs} = \underline{\Psi}_{asd} + \underline{\Psi}_{asi} + j\underline{\Psi}_{b1d} + j\underline{\Psi}_{b1i} + j\underline{\Psi}_{b2d} + j\underline{\Psi}_{b2i} = \\ &= \frac{\Psi_{as}}{\sqrt{2}}\left[e^{j(\omega_s t-\pi/2)} + e^{-j(\omega_s t-\pi/2)}\right] + \frac{\Psi_{b1}}{\sqrt{2}}\left[e^{j(\omega_s t-\pi/2)} + e^{-j(\omega_s t+\pi/2)}\right] + \frac{\Psi_{b2}}{\sqrt{2}}\left[e^{j\omega_s t} + e^{-j(\omega_s t-\pi)}\right] = \\ &= \frac{\Psi_{as}}{\sqrt{2}}\left[\lambda\sin\varepsilon - j(1+\lambda\cos\varepsilon)\right]e^{j\omega_s t} + \frac{\Psi_{as}}{\sqrt{2}}\left[-\lambda\sin\varepsilon + j(1-\lambda\cos\varepsilon)\right]e^{-j\omega_s t} = \underline{\Psi}_{sRd} + \underline{\Psi}_{sRi}\end{aligned} \tag{13}$$

Thus, is analytically proved that, in contrast to symmetrical condition that keep nothing but one component (forward or backward), the unsymmetrical condition has two flux components: a forward one, more significant in amplitude and a weaker backward component (this fact is generally valid for the studied quantities). The two components can be expressed as follows:

$$\begin{aligned}\underline{\Psi}_{sRd} &= Ae^{j(\omega_s t+\alpha_d)};\ A = \frac{q_1\Psi_{as}}{\sqrt{2}};\ q_1 = \sqrt{1+\lambda^2+2\lambda\cos\varepsilon};\quad \alpha_d = \arctan\frac{1+\lambda\cos\varepsilon}{-\lambda\sin\varepsilon} \\ \underline{\Psi}_{sRi} &= Be^{-j(\omega_s t-\alpha_i)};\ B = \frac{q_2\Psi_{as}}{\sqrt{2}};\ q_2 = \sqrt{1+\lambda^2-2\lambda\cos\varepsilon};\ \alpha_i = 180^0 - \arctan\frac{1-\lambda\cos\varepsilon}{\lambda\sin\varepsilon}\end{aligned} \tag{14}$$

The next step is a rotation of the space-time phase vector inside the complex space with the angle $-\alpha = -(\alpha_i + \alpha_d)/2$. This operation is equivalent with a multiplication with $e^{-j(\alpha_d+\alpha_i)/2}$. It results:

$$\underline{\Psi}_{sR}e^{-j\alpha} = Ae^{j\left(\omega_s t - \frac{\alpha_i-\alpha_d}{2}\right)} + Be^{-j\left(\omega_s t - \frac{\alpha_i-\alpha_d}{2}\right)} \tag{15}$$

The trigonometric form is:

$$\Psi_{sR}e^{j\varphi_{sR}}\cdot e^{-j\alpha} = (A+B)\cos[\omega_s t - (\alpha_i-\alpha_d)/2] + j(A-B)\sin[\omega_s t - (\alpha_i-\alpha_d)/2] \tag{16}$$

Dissociation of the real (*x* subscript) and imaginary (*y* subscript) parts lead to the equation system:

$$\begin{aligned}\Psi_{sRx}\cos\alpha + \Psi_{sRy}\sin\alpha &= (A+B)\cos[\omega_s t - (\alpha_i-\alpha_d)/2] = \Psi_{RX} \\ -\Psi_{sRx}\sin\alpha + \Psi_{sRy}\cos\alpha &= (A-B)\sin[\omega_s t + (\alpha_i-\alpha_d)/2] = \Psi_{RY}\end{aligned} \tag{17}$$

which, by eliminating the parameter *t*, gives;

$$\text{(E)}\quad \frac{\Psi_{RX}^2}{(A+B)^2} + \frac{\Psi_{RY}^2}{(A-B)^2} = 1 \tag{18}$$

In conclusion, the apex coordinates of the **representative flux** vector which are related to the new axes (**XOY**) and rotated by the angle $\alpha = (\alpha_i + \alpha_d)/2$ towards the initial system (**xOy**) are accomplishing the characteristic equations of an ellipse (E) with *A*+*B* and *A*-*B* as semi-axes.

The apex of the *forward* vectors covers a circle (C_d), while the backward components cover the circle (C_i). Bothe vectors have coequal angular speeds, ω_s, but to opposite directions. Fig. 2 presents the representative phase vectors of the total fluxes corresponding to three consecutive moments: : $\omega t = 0$, - the apex in A; $\omega t = \pi/4$, - the apex in B; $\omega t = \pi/2$, - the apex in C. As an example, using the values $\lambda = 0.75$; $\varepsilon = \pi/6$; $\Psi_{as} = 6$, then

$$\underline{\Psi}_{sR} = 4.2\left[e^{j(\omega_s t-\pi/2)} + e^{-j(\omega_s t-\pi/2)}\right] + 2.8\left[e^{j(\omega_s t-\pi/2)} + e^{-j(\omega_s t+\pi/2)}\right] + 1.6\left[e^{j\omega_s t} + e^{-j(\omega_s t-\pi)}\right] \tag{19}$$

Separation of the forward and backward terms gives

$$\underline{\Psi}_{sR} = (1.6 - j7)e^{j\omega_s t} + (-1.6 + j1.4)e^{-j\omega_s t} = 7.2e^{j(\omega_s t - 77^0)} + 2.1e^{-j(\omega_s t - 137^0)};\ \alpha = 30^0 \tag{20}$$

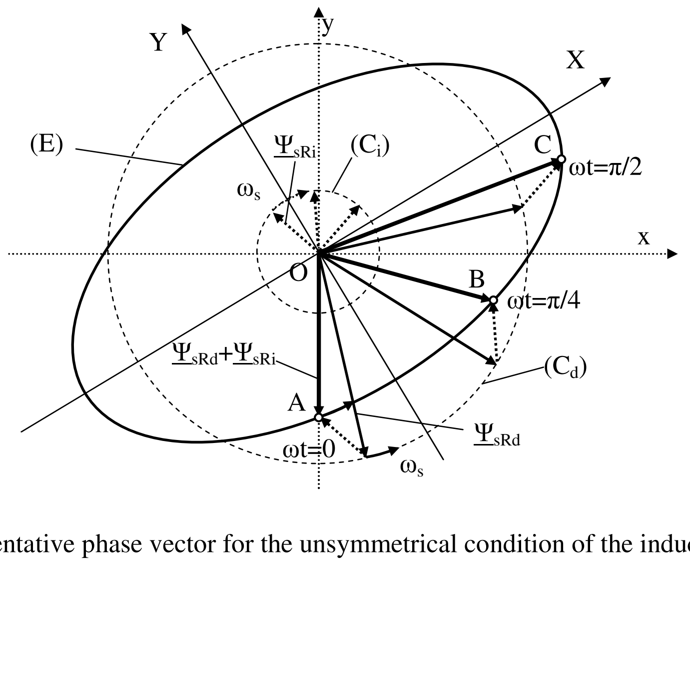

Fig. 2 Representative phase vector for the unsymmetrical condition of the induction machine

The amplitude of total flux during a period cover the range (9.3 - 5.1)Wb. It is obvious that the instantaneous angular speed of the resultant flux is a variable quantity despite the assumption which asserts that the mean value remains constant and coequal with the synchronism one. Practically, during a period, the torque has twice both maximum and minimum values. This may cause intolerable vibrations and noise or possible mechanical faults. These variable torques determine non-uniform rotor speed, increased frictions, higher temperature irradiated non-uniformly in the machine components, trepidations, premature ageing of the bearings, a global decrease of the output power and lifetime.

IV. ANALYSIS OF UNSYMMETRICAL CONDITION USING REPRESENTATIVE SPACE-TIME PHASE VECTORS

A proper analysis can be achieved as follows. One starts with the equations of two-phase unsymmetrical induction machine written in the simplified complex manner, under matrix form, [15-20]:

$$\begin{bmatrix} \underline{U}_{as} \\ \underline{U}_{bs} \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} \nu_{rs}+j\omega_s & 0 & 0 & -\nu_{hs} \\ 0 & \nu_{rs}+j\omega_s & -\nu_{hs} & 0 \\ 0 & -\nu_{hr} & \nu_{sr}+j\omega_s & -\omega_R \\ -\nu_{hr} & 0 & \omega_R & \nu_{sr}+j\omega_s \end{bmatrix} \cdot \begin{bmatrix} \underline{\psi}_{as} \\ \underline{\psi}_{bs} \\ \underline{\psi}_{qr} \\ \underline{\psi}_{dr} \end{bmatrix} \tag{21}$$

Assuming the superposing effect principle for voltages and total fluxes, one can formulate the connective expression between complex quantities and corresponsive space-time phase vectors as follows

$$\underline{U}_{sR} = \underline{U}_{as} + j\underline{U}_{bs}; \underline{\psi}_{sR} = \underline{\psi}_{as} + j\underline{\psi}_{bs}; \underline{\psi}_{rR} = \underline{\psi}_{dr} + j\underline{\psi}_{qr} \tag{22}$$

In matrix equation (21) one multiplies the 2nd and 3rd rows with *j*, which has as consequence the modification of the terms including ω_R :

$$\begin{bmatrix} \underline{U}_{as} \\ j\underline{U}_{bs} \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} \nu_{rs}+j\omega_s & 0 & 0 & -\nu_{hs} \\ 0 & \nu_{rs}+j\omega_s & -\nu_{hs} & 0 \\ 0 & -\nu_{hr} & \nu_{sr}+j\omega_s & -j\omega_R \\ -\nu_{hr} & 0 & -j\omega_R & \nu_{sr}+j\omega_s \end{bmatrix} \cdot \begin{bmatrix} \underline{\psi}_{as} \\ j\underline{\psi}_{bs} \\ j\underline{\psi}_{qr} \\ \underline{\psi}_{dr} \end{bmatrix} \tag{23}$$

The system equation (23) leads to other two equations: one for the representative space-time phase vector of the stator total flux and the other for the similar rotor phase vector. The analysis implies two steps.

a) One consider a first machine, denoted with F.M., which include nothing but rotational representative phase vectors that that circulate forward (denoted with *d* subscript). The following notations are used:

$$\underline{N}_{rs} = \nu_{rs} + j\omega_s; \underline{N}_{sr} = \nu_{sr} + j\omega_s;$$
$$\nu_{tt}\omega_s = \nu_{sr}\nu_{rs} - \nu_{hs}\nu_{hr}; \omega_s - \omega_R = s\omega_s .$$

The first equation comes from summation of the first two rows and the second equation is the sum of the third and forth rows. The matrix form of the matrix is:

$$\text{(F.M.)}\begin{bmatrix} \underline{U}_{sRd} \\ 0 \end{bmatrix} = \begin{bmatrix} \underline{N}_{rs} & -\nu_{hs} \\ -\nu_{hr} & \underline{N}_{sr} - j\omega_R \end{bmatrix} \times \begin{bmatrix} \underline{\psi}_{sRd} \\ \underline{\psi}_{rRd} \end{bmatrix} \tag{24}$$

The right member determinant is a complex quantity:

$$\begin{aligned} \underline{\Delta} &= \omega_s \left[(\nu_{tt} - s\omega_s) + j(\nu_{sr} + s\nu_{rs}) \right]; \Delta^2 = \\ &= \omega_s^2 \left[(\nu_{rs}^2 + \omega_s^2) s^2 + 2s\nu_{hs}\nu_{hr} + \nu_{sr}^2 + \nu_{tt}^2 \right] \end{aligned} \tag{25}$$

The representative space-time phase vectors of the fluxes are:

$$\underline{\psi}_{sRd} = \frac{\underline{U}_{sRd}}{\Delta^2}(\nu_{sr} + js\omega_s)\underline{\Delta}^*; \underline{\psi}_{rRd} = \frac{\underline{U}_{sRd}}{\Delta^2}\nu_{hr}\underline{\Delta}^*; \tag{26}$$

The electromagnetic torque developed by the forward machine is:

$$\begin{aligned} M_{ed} &= -\frac{p}{L_{\sigma\sigma}} \mathrm{Re}\left(j\underline{\psi}_{sRd} \underline{\psi}_{rRd}^* \right) = \\ &= \frac{p\nu_{hr}}{\omega_s L_{\sigma\sigma}} U_{sRd}^2 \frac{s}{(\nu_{rs}^2 + \omega_s^2)s^2 + 2s\nu_{hs}\nu_{hr} + \nu_{sr}^2 + \nu_{tt}^2} \end{aligned} \tag{27}$$

Observation: A reversal of the supply phase order (or any polarity phase inversion) with preservation of rotor rotation direction determine the reversal of the rotation direction of the representative phase vector of applied voltages and fluxes. This fact implies the reversal of the sign of the pulsation, $\omega_s \to -\omega_s$. In (24) act the new parameters $\underline{N}_{rs}^* = \nu_{rs} - j\omega_s; \underline{N}_{sr}^* = \nu_{sr} - j\omega_s$. The machine operates as brake and the new equations give easily the torque expression.

b) From the viewpoint of the backward components, the machine acts as a brake (B.M.), according to equations:

$$\begin{bmatrix} \underline{U}_{sRi} \\ 0 \end{bmatrix} = \begin{bmatrix} \underline{N}_{rs}^* & -\nu_{hs} \\ -\nu_{hr} & \underline{N}_{sr}^* - j\omega_R \end{bmatrix} \times \begin{bmatrix} \underline{\psi}_{sRi} \\ \underline{\psi}_{rRi} \end{bmatrix} \tag{28}$$

where $-\omega_s - \omega_R = -(2-s)\omega_s$.

The left member determinant is:

$$\begin{aligned} \underline{\Delta}_1 &= \omega_s \left[(\nu_{tt} - (2-s)\omega_s) + j(\nu_{sr} + (2-s)\nu_{rs}) \right]; \\ \Delta_1^2 &= \omega_s^2 \left[A(2-s)^2 + 2B(2-s) + C \right] \end{aligned} \tag{29}$$

where: $A = \nu_{rs}^2 + \omega_s^2; B = \nu_{hs}\nu_{hr}; C = \nu_{sr}^2 + \nu_{tt}^2$

The representative space-time phase vectors of the fluxes are:

$$\underline{\psi}_{sRi} = \frac{\underline{U}_{sRi}}{\Delta_1^2}\left[\nu_{sr} - j(2-s)\omega_s \right]\underline{\Delta}_1^*; \underline{\psi}_{rRi} = \frac{\underline{U}_{sRi}}{\Delta_1^2}\nu_{hr}\underline{\Delta}_1^* \tag{30}$$

The electromagnetic torque developed by the "backward machine" is:

$$M_{ei} = -\frac{p}{L_{\sigma\sigma}}\mathrm{Re}\left(j\underline{\psi}_{sRi}\underline{\psi}^{*}_{rRi}\right) = -\frac{p\nu_{hr}}{\omega_s L_{\sigma\sigma}}U^2_{sRi}\frac{2-s}{A(2-s)^2+2Bs+C} \tag{31}$$

The resultant torque obtained by means of superposing effect law, is:

$$M_{erez} = M_{ed} + M_{ei} = \frac{p\nu_{hr}}{\omega_s L_{\sigma\sigma}}\left[\frac{sU^2_{sRd}}{As^2+2Bs+C} - \frac{(2-s)U^2_{sRi}}{A(2-s)^2+2B(2-s)+C}\right] \tag{32}$$

The components of the representative phase vector of the voltages can be similarly deduced:

$$\begin{aligned}&\underline{U}_{sR} = \underline{U}_{as} + j\underline{U}_{bs} = \underline{U}_{asd} + \underline{U}_{asi} + j(\underline{U}_{bsd} + \underline{U}_{bsi}) = (1+\underline{\lambda}_d)\underline{U}_{asd} + (1-\underline{\lambda}_i)\underline{U}_{asi} = \\ &(1+\lambda\cos\varepsilon + j\lambda\sin\varepsilon)\underline{U}_{asd} + (1-\lambda\cos\varepsilon + j\lambda\sin\varepsilon)\underline{U}_{asi} = \frac{U_{as}}{\sqrt{2}}\left[E_d e^{j\varepsilon_d}e^{j\omega_s t} + E_i e^{j\varepsilon_i}e^{-j\omega_s t}\right] = \underline{U}_{sRd} + \underline{U}_{sRi}\end{aligned} \tag{33}$$

where: $E_d = \sqrt{1+\lambda^2+2\lambda\cos\varepsilon}; E_i = \sqrt{1+\lambda^2-2\lambda\cos\varepsilon}; \varepsilon_d = arctg\frac{\lambda\sin\varepsilon}{1+\lambda\cos\varepsilon}; \varepsilon_i = arctg\frac{\lambda\sin\varepsilon}{1-\lambda\cos\varepsilon}$

Hence, the resultant torque

$$M_{erez} = M_{ed} + M_{ei} = \frac{p\nu_{hr}U^2_{as}}{2\omega_s L_{\sigma\sigma}}\left[\frac{s(1+\lambda^2+2\lambda\cos\varepsilon)}{As^2+2Bs+C} - \frac{(2-s)(1+\lambda^2-2\lambda\cos\varepsilon)}{A(2-s)^2+2B(2-s)+C}\right] \tag{34}$$

For particular case one obtain:

- Machine with symmetrical supply system - $\lambda = 1; \varepsilon = 0; E_d = 2; E_i = 0; \varepsilon_d = \varepsilon_i = 0;$

$$M_{erez} = M_{ed} = \frac{2p\nu_{hr}U^2_{as}}{\omega_s L_{\sigma\sigma}}\frac{s}{As^2+2Bs+C} \tag{35}$$

- Machine with single-phase supply system (broken *b-y* phase) - $\lambda = 0; \varepsilon = 0; E_d = 1; E_i = 1; \varepsilon_d = \varepsilon_i = 0;$

$$M_{erez} = M_{ed} + M_{ei} = \frac{p\nu_{hr}U^2_{as}}{2\omega_s L_{\sigma\sigma}}\left[\frac{s}{As^2+2Bs+C} - \frac{(2-s)}{A(2-s)^2+2B(2-s)+C}\right] \tag{36}$$

Obviously, for $s=1$ (start-up) one obtain $M_{erez}=0$. At same the time, it is noticeable that the dependence $M_{erez}=f(s)$ is a symmetric curve around start-up point, $M_{erez}(1-x)=-M_{erez}(1+x)$.

- Machine with both windings connected to the same voltage (null difference of phase between the two applied voltages), $\lambda = 1; \varepsilon = \pi/2; E_d = E_i = \sqrt{2}; \varepsilon_d = \varepsilon_i = \pi/4$. As a matter of fact, this is a *complete single phase or 1/1 machine*. The torque expression is:

$$M_{erez} = M_{ed} + M_{ei} = \frac{p\nu_{hr}U^2_{as}}{\omega_s L_{\sigma\sigma}}\left[\frac{s}{As^2+2Bs+C} - \frac{(2-s)}{A(2-s)^2+2B(2-s)+C}\right] \tag{37}$$

There is a duplication of the torque for a certain slip. This is a solid argument for the solution that uses for the single phase motors only 2/3 slots and consequently the developed torque decrease to 8:9 value. As expected, for $s=1$ then $M_{erez}=0$.

No doubt, there is a confirmation of these results which are similar to ones that are obtained by means of other methods such as "symmetrical components method" [21-22].

Graphic inference of the amplitude of the two voltage components is presented in Fig. 3. The graphic construction is made for a specific case, and more precisely at $t=0$. After the calculus of the length of the two voltage components (forward and backward) and their difference of phase corresponding to $t=0$, one determine the track of the apices representing a circle. Each component length represents the circle radius, which runs to opposite directions with coequal speeds, ω_s.

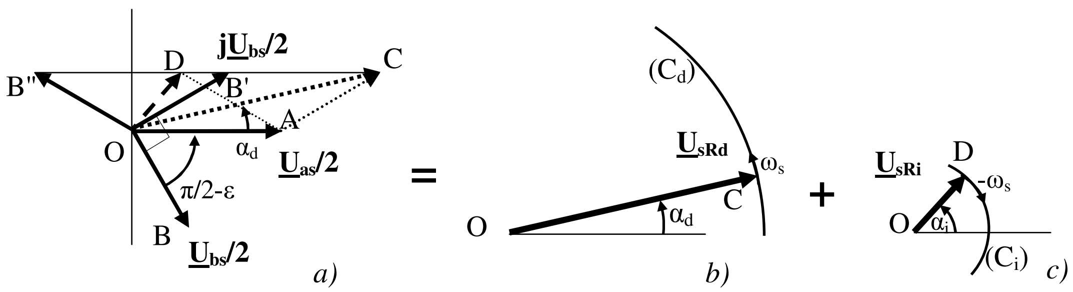

Fig. 3 Graphic inference of the representative space-time phase vector component
a) Two-phase unsymmetrical voltage system, b) Forward rotational phase vector, c) Backward rotational phase vector

The following methodology is used (Fig. 3). The voltage $\underline{U}_{as}$ is considered as phase reference, Fig. 3a. One plot the phase vector $\underline{OA}$, its value being half voltage amplitude, u_{as}, that is $|OA| = \frac{U_{as}}{\sqrt{2}}$. Similarly, one plot the phase vector $\underline{OB}$ with $|OB| = \frac{U_{bs}}{\sqrt{2}}$ but with a difference of phase of $\varepsilon-\pi/2$ (which is negative for this case). Further, the phase vector $\underline{OB}$ is rotated towards positive direction with $\pi/2$ resulting $\underline{OB}'$. The vectors $\underline{OA}$ cu $\underline{OB}'$ are geometrically summated resulting $\underline{OC} = \underline{U}_{sRd}$, that is *the representative forward rotational phase vector*. Then the vector $\underline{OB}''$ is obtained as the symmetric segment against vertical axis. The sum of $\underline{OB}''$ with $\underline{OA}$ gives $\underline{OD} = \underline{U}_{sRi}$, that is *the representative backward rotational phase vector*. The representative forward rotational phase vector covers the circle (C_d) towards positive trigonometric direction (Fig. 3b) and the representative backward rotational phase vector covers the circle (C_i) towards negative trigonometric direction (Fig. 3c). Obviously, the apex of the representative rotational phase vector covers an ellipse. This graphical construction is justified by the following reasoning. The cosine theorem applied in OAC triangle gives $|OC| = \left(U_{as}/\sqrt{2}\right)\sqrt{1+\lambda^2+2\lambda\cos\varepsilon}$ and for OAD triangle $|OD| = \left(U_{as}/\sqrt{2}\right)\sqrt{1+\lambda^2-2\lambda\cos\varepsilon}$. These are the lengths of the forward and backward phase vectors.

V. CONCLUSION

The representative space-time rotational phase vectors of total fluxes represents a useful tool for understanding the phenomena that take place inside the induction machine (with stator-inductor, rotor-armature). They give a physical signification close to image of the traveling waves. The equations have a reduced number of variables. Practically, there are only voltages (that characterize the electric field) and total fluxes (characterizing the magnetic field). The presence of current is no longer necessary.

The equations containing *nothing but fluxes* lead to simple analytical expressions for total fluxes of the stator and rotor. It is easy to handle these equations both for the analysis of symmetrical and unsymmetrical conditions.

For symmetrical condition, the apex of the representative stator and rotor phase vectors (for flux) covers a circle and the rotation speeds are constant. For unsymmetrical condition, the apices cover ellipses and the instantaneous speeds during a revolution vary between two limits. The analysis can be accomplished by using two representative phase vectors: a forward and a backward one, respectively. They have coequal but opposite directions.

For unsymmetrical condition is possible to have a significant saturation of the magnetic circuit corresponding to major axis position. In this approach, this fact can be easier pointed out in comparison with classic formulations where the presence of currents is mandatory.

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